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TOPICAL ISSUE

Determination of the Relative Positionong Based on Magnetic Gradiometry Measurements

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Abstract—The paper is devoted to solving the problem of determining the relative spatial arrangement and orientation of objects. The task was set: to show the fundamental possibility of spatial and angular relative positioning when using the parameters of the magnetic field gradient in tensor form and in the form gradient of an absolute value vector as measurement information for the magnetic field of a local dipole source. The solution of the problem is presented along with the features and limitations for both forms of representation are considered. The principles of construction of magnetic gradiometry measurement systems are briefly described, the limitations of technical implementation are considered, and the benefits of using an alternating magnetic field source is outlined. The results of modelling are presented, proving the possibility of using the proposed positioning method for various engineering problems.

Keywords: relative positioning, magnetic field vector, magnetic field gradient, gradient tensor

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1. INTRODUCTION

The solution to many engineering problems is in one way or another connected with the need to determine the relative position of objects during their interaction. For example, high-precision control is necessary to control movement during mid-air refueling; mooring a vessel to a pier, loading terminal or drilling platform; landing an aircraft on a limited area, docking spacecraft and underwater vehicles [1–3]. Solving the relative positioning problem assumes that in the coordinate system associated with one of the objects, it is necessary to determine the radius vector of the location point of another object, as well as their mutual angular orientation. Usually, to solve such problems, gyroinertial systems, multi-antenna GNSS receivers, optical systems, etc. are used, but in many cases the technical solution is significantly complicated by the special aspects of the application conditions, thus becoming excessively large. In many cases, positioning accuracy is insufficient. The possibility of using magnetic gradient measurements provides possibility to develop relative positioning methods. The basis of the idea is the fact that the direction and absolute magnitude of the magnetic field vector of a point dipole transmitter at a certain point in space is fully determined by the magnitude and direction of the source dipole magnetic moment vector and the position of the measurement point. A type of the dependence of the field strength allows, from the data obtained by receiver of the local transmitter, to simultaneously determine their relative spatial and angular location.

2. PROBLEM STATEMENT

Let a local dipole magnetic field transmitter with an arbitrary direction of the dipole magnetic moment vector $\mathbf{M}$ be located at the origin and let the field be measured at an arbitrary point in space determined by the radius vector $\mathbf{r}$ in this coordinate system (Fig. 1).

For the magnetic field potential U^B of a local dipole transmitter in the associated coordinate system, the following relation is valid:

$$U^B = \frac{\mu\mu_0 \mathbf{r}^{\mathrm{T}}\mathbf{M}}{4\pi\left(\mathbf{r}^{\mathrm{T}}\mathbf{r}\right)^{3/2}}. \tag{1}$$

Here μ and μ_0 are the magnetic permeability of the medium and the magnetic constant in the international system of units (SI), respectively.

Differentiating (1), we obtain the values for the field vector:

$$\nabla U^B = \frac{\mu\mu_0\left|\mathbf{M}\right|}{4\pi|\mathbf{r}|^5}\begin{pmatrix} 3y_1^2 - |\mathbf{r}|^2 \\ 3y_1y_2 \\ 3y_1y_3 \end{pmatrix}, \tag{2}$$

and gradient tensor:

$$\mathbf{U} = \nabla\nabla^{\mathrm{T}}U^B = \frac{3\mu\mu_0\left|\mathbf{M}\right|}{4\pi|\mathbf{r}|^7}\begin{pmatrix} -2y_1^3 + 3y_1y_2^2 + 3y_1y_3^2 & -4y_1^2y_2 + y_2^3 + y_2y_3^2 & -4y_1^2y_3 + y_2^2y_3 + y_3^3 \\ -4y_1^2y_2 + y_2^3 + y_2y_3^2 & y_1^3 - 4y_1y_2^2 + y_1y_3^2 & -5y_1y_2y_3 \\ -4y_1^2y_3 + y_2^2y_3 + y_3^3 & -5y_1y_2y_3 & y_1^3 + y_1y_2^2 - 4y_1y_3^2 \end{pmatrix}. \tag{3}$$

The parameters of the magnetic field at the observation point are determined by a tensor gradiometric receiver, the coordinate system of which is oriented arbitrarily relative to the field source. Determining the parameters of the gradient tensor consists of measurement the field values at several points in space near the point with the radius vector $\mathbf{r}$ [5].

The problem is by knowing the magnitude and direction of the vector of the dipole magnetic moment of the field source in the coordinate system associated with it, and also having the results of parameter measurement of the field gradient tensor in the area where the observation point is located in the coordinate system of the receiver to determine the parameters of the radius vector between the source and the field receiver, as well as the direction of the vector of the dipole magnetic moment of the transmitter in the coordinate system of the receiver.

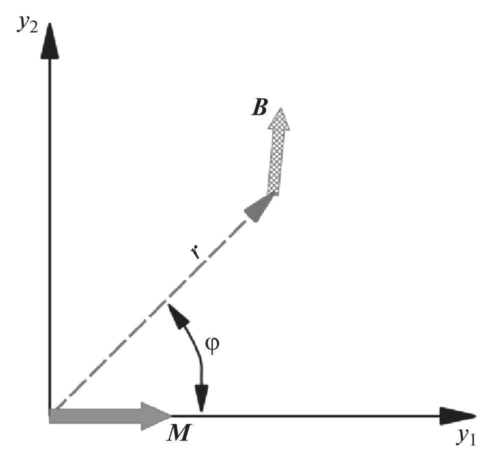

Fig. 1. The vector of the dipole magnetic moment and the vector of field in the coordinate system associated with the transmitter.

3. POSITIONING BASED ON MAGNETIC FIELD GRADIENT TENSOR MEASUREMENTS

The tensor parameters (3), which are important for solving the positioning problem, can be obtained by measurement the field at spatially separated points, but close enough as compared to the distance to the transmitter (so that one can limit ourselves to a linear approximation of the dependence of the field change on the distance).

Magnetic field potential is a harmonic function. Therefore, the tensor (3) is symmetric, and its trace is equal to zero. Thus, it contains not nine, but only five independent components. Moreover, by orthogonal transformations the coordinate system of the transemitter can be brought to the principal axes of the tensor. In this system, only its diagonal elements are nonzero.

The angular divergence α of the coordinate systems of the principal axes of the tensor y' and the y system is determined by the angle φ between the vector of the dipole magnetic moment and the radius vector **r** (Fig. 2). Knowing the angle φ between the radius vector **r** and the direction of the vector **M** from (3) it follows that the values of the angles α and φ are related to the relations of the main components of the tensor (Fig. 3). It also follows from (3) that when the coordinate

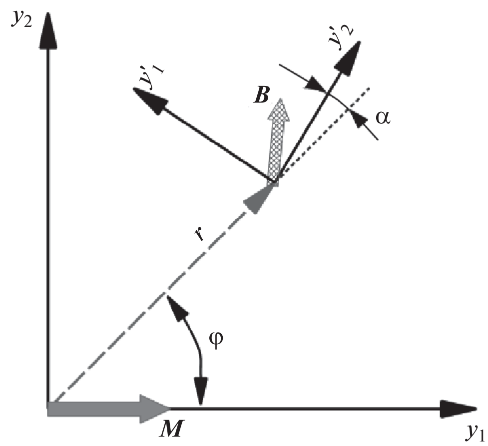

Fig. 2. To the parameters of the field gradient tensor of a point dipole: the principal axes of the tensor at the field measurement point.

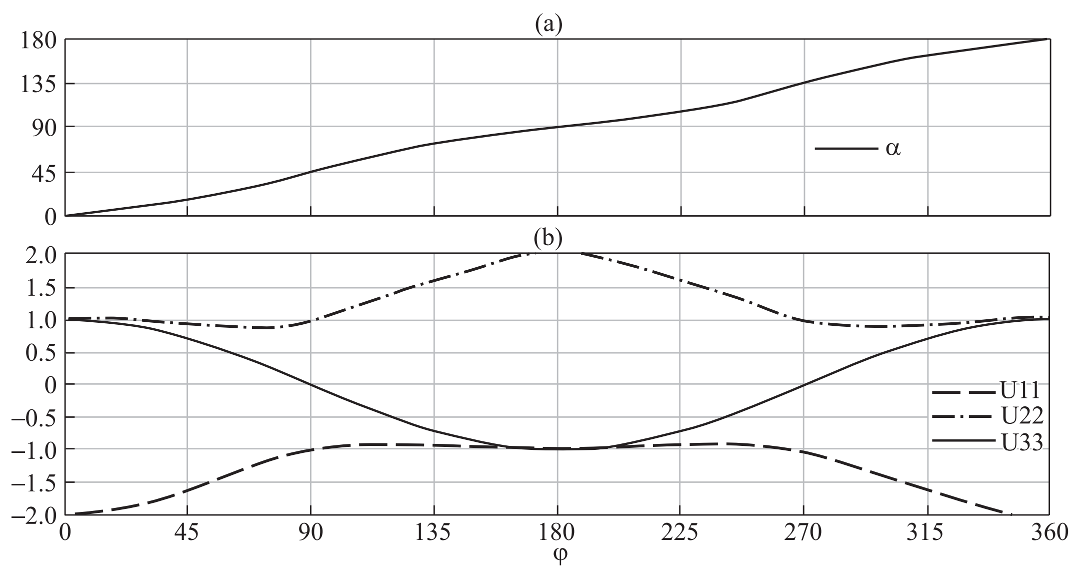

Fig. 3. Dependence of the tensor parameters on the value of the angle φ: (a) angular divergence of coordinate systems (angle α), (b) values of the main components of the tensor $\mathbf{U}'$.

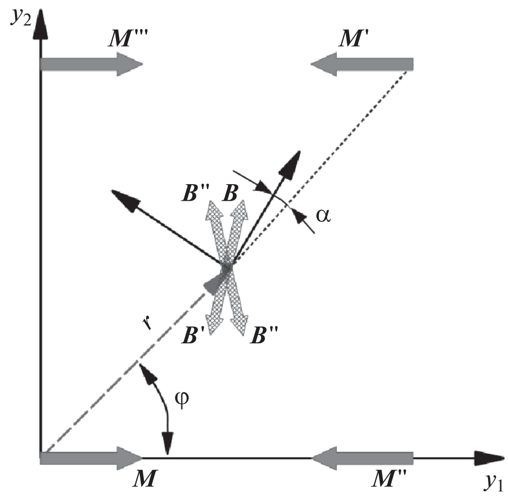

Fig. 4. Uncertainty in determining the DMM from measurements of the magnetic field gradient tensor.

system of the receiver is rotated around the y_3 axis by 180°, the tensor value remains unchanged, only its main components change places and change sign. Figure 3 clearly shows this.

The fact that the values of the angles α and φ are determined by the same ratios of the values of the main components of the tensor (the angle α is determined up to rotation for 180°) gives grounds for determining the directions of the radius from the data of magnetic gradiometry measurements vector **r** and dipole moment vector **M**. With a known absolute value of the dipole magnetic moment, the distance between the transmitter and the receiver can be determined, which constitutes the solution to the relative positioning problem.

Unfortunately, the solution to the positioning problem is ambiguous. Having the results of measurements of the components of the tensor $\mathbf{U}'$ at a certain point in space, the problem of positioning the dipole-transmitter in the system of the principal axes of the tensor can be considered as follows:

On the interval from 0° to 90° in φ, the $\mathbf{U'_{11}}$ component, corresponding to the value of the second derivative with respect to the first component, is maximum in amplitude and negative. Having determined direction of the first axis, it is necessary to choose the direction of the third so that the minimum amplitude of the gradient corresponds to it. The second axis complements the vector tripod to the right.

On the interval from 90° to 180° in φ, the component $\mathbf{U'_{22}}$, corresponding to the second derivative with respect to the second component, is maximum in amplitude and positive. Having determined direction of the second axis, it is necessary to choose the direction of the third axis so that the minimum amplitude of the gradient corresponds to it. The direction of the first axis should set the right tripod.

In the interval from 360° to 180° in φ the tensor components behave in the same way as in the interval from 0° to 180°. Thus, the angle φ can only be determined up to sign. Moreover, if for φ from 0° to 180° the angle α is determined, then for φ from 360° to 180° this angle is α.

Due to their insensitivity to 180° rotation, the components of the gradient tensor determine two possible directions of the location of the transmitter dipoles that could create the measured gradient—$\mathbf{M}$ and $\mathbf{M}'$. These possible transmitters are located opposite to the observation point, identical in size and opposite in direction. In addition, two more dipoles $\mathbf{M}''$ and $\mathbf{M}'''$ also correspond to the measurement results due to symmetry with respect to the dipole axis (Fig. 4).

Thus, the problem of determining the position of the transmitter from measurements of the gradient tensor is uniquely solved only if the quadrant of its location is known. It is also clear from Fig. 4 that additional information about the direction cosines of the field vector **B** will allow us to immediately reject incorrect hypotheses, and if we assume that the absolute value of the dipole magnetic moment of the transmitter is known, then according to (2) and (3) we can also determine the distance to the dipole, i.e. obtain the necessary information to solve the relative positioning problem.

Note, however, that the result of measurement the parameters of the field gradient tensor of a point transmitter is invariant to the rotation of the coordinate system associated with the field source around an axis whose direction coincides with the direction of the dipole magnetic moment vector. This means that the measurements taken are not enough to determine the relative angular orientation of objects.

To solve this problem, additional information can be used, which for some conditions is quite natural. Thus, when the ship approaches the berth, the directions of the vertical axes in the systems associated with the field source and the transemitter can be considered coincident. If the field source is located on the cone of the refueling hose, and the dipole moment vector is directed along it, then the effect of rotating the coordinate system around the moment vector does not change anything from the point of view of the docking process during air refueling.

A complete solution to the positioning problem can be obtained by placing not one, but several dipole transmitters on one of the interacting objects. The technical capacity of performing correct measurements in this option is discussed below.

4. POSITIONING USING A VECTOR MAGNETOGRADIOMETER

It is important to note that at the hardware level, measurement tensor components (3) involves the use of three spatially separated vector sensors—field induction meters. Today, such devices are characterized by low accuracy rates.

Scalar magnetically sensitive sensors that directly measure the absolute value of the field induction are somewhat more accurate. Their operation is based on the quantum effects of precession of atoms in polarized light (optically pumped quantum magnetometer) or protons (proton and Overhauser magnetometers) [6]. In this regard, it is interesting to consider the possibility of determining the spatial location and orientation of the field source based on the results of determining the gradient vector of the absolute value of the magnetic field induction vector. The components of this vector can be measured by a system composed of four spatially separated scalar sensors. The value of the gradient vector of the absolute value of the field and the gradient tensor are related by the equation

$$\nabla |\mathbf{B}| = \left(\nabla \mathbf{B}^{\mathrm{T}}\right) (\mathbf{B}/ |\mathbf{B}|) . \tag{4}$$

This equation is obtained by differentiating $|\mathbf{B}| = \sqrt{\mathbf{B}^{\mathrm{T}}\mathbf{B}}$. It turns out that to solve the positioning problem using vector gradiometry data during measurements, it is necessary to determine not only the scalar field values at four points, but also the direction of the field vector (the ratio $\mathbf{B}/|\mathbf{B}|$ in (4)).

Calculations show that the gradient vector is directed predominantly towards the source (Fig. 5). The magnitude of the angular discrepancy β between the gradient vector and the direction to the transmitter depends on the angle φ between the directions of the radius vector $\mathbf{r}$ and the dipole moment vector $\mathbf{M}$. The maximum discrepancy is about 15°.

The dependence of the angular divergence of the β radius vector and the gradient vector on the direction to the dipole is shown in Fig. 6. It is clearly seen that even with direct measurements of

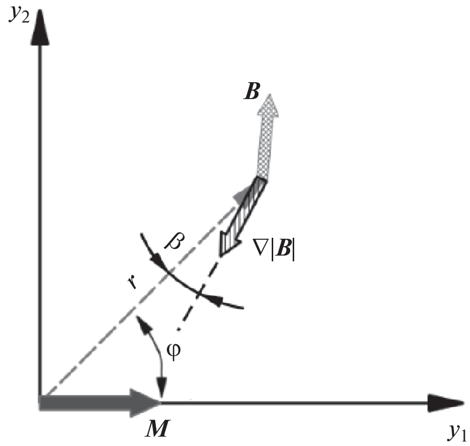

Fig. 5. Measurement of the field and gradient vector of the absolute value of the magnetic vector.

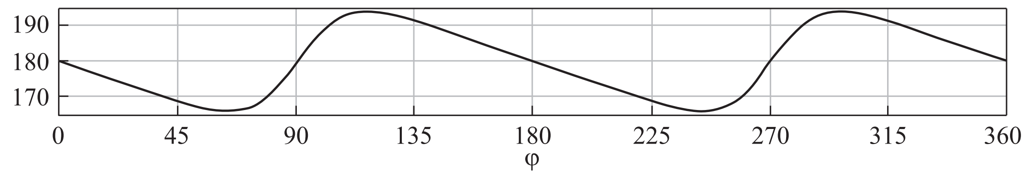

Fig. 6. Divergence of the directions of the radius vector and the gradient vector.

the gradient vector, the problem of determining the direction to the dipole-transmitter is solved, although roughly, but without the ambiguity inherent in tensor measurements.

From the measurement data of the vector with a known value of the dipole magnetic moment, the distance to the transmitter can be calculated, but additional information is needed to determine the radius vector. This additional information can be obtained from a series of measurements as objects move relative to each other. It is also possible to use readings from several spaced gradiometers. Since the symmetry axes of equivalent solutions are lines drawn through the measurement point parallel and perpendicular to the dipole axis, then for three gradiometers that do not lie on the same line, the result of determining the source position will be a single point. Note that such a scheme, although technically complex, does not require specifying the value of the dipole moment of the source, i.e. makes it possible not only to get rid of ambiguity, but also to localize the source, while determining the value of its dipole moment.

5. LIMITATIONS OF TECHNICAL IMPLEMENTATION

The choice of the form of presentation of magnetic gradient information, and therefore the method of measurement, and the structure of the magnetic measurement installation for solving the problem of relative positioning is largely determined by the working conditions. A significant role is played by the functioning of sensors, the dynamics of object movement, the presence of interference in the application area, and much more. However, it is important that in addition to the field caused by the operation of an artificial dipole-transmitter, the receiver inevitably registers the natural magnetic field of the earth, which is very large in magnitude, usually has a significant gradient, and also unpredictably changes in time under the influence of natural geomagnetic disturbances.

This fact, however, should not be considered a significant hindrance to the implementation of the methods and algorithms discussed above, since an inductor (loop dipole) powered by an alternating current of a certain shape can be used as a field source. This approach allows the use of a two-and three-dipole transmitter, thereby overcoming the ambiguity in determining the direction to the source in the case of using a tensor receiver. The task of isolating the field vector of each transmitter individually is not significantly difficult.

Another kind of difficulty in the application of the considered algorithms turns out to be related to the perfomance features of magnetically sensitive sensors and, first of all, the influence of magnetic interference during the measurement process. The use of an alternating magnetic field allows the use of narrowband filtering algorithms, which significantly reduces this negative impact. Moreover, this approach allows the use of induction magnetometers as recievers, which are not capable of measurement the constant component of the field, but have a significantly higher sensitivity in relation to other types of sensors.

It is also important to note that the considered algorithms are basic and do not take into account fundamentally important aspects of a possible technical implementation. Thus, the field source is assumed to be local, or more precisely, a point dipole transmitter. However, a technically feasible transmitter inevitably has a non-zero size, and therefore its field differs from the field of an ideal dipole. The degree of difference decreases with distance, however, with a significant distance, the amplitude of the measured field decreases significantly, the limitations of the sensitivity and accuracy of the sensors, and the negative influence of various external interferences are fully manifest themselves.

Similar difficulties in technical implementation are typical for gradient field receivers. The definition of the gradient as the second derivative of the potential assumes that the increments of the field induction vector along the selected directions are measured at a point at infinitesimal distance increments. In the technical implementation, even at small distances between the field measurement points, the discrepancy between the values of the derivative $\frac{\partial \mathbf{B}}{\partial x}$ and the ratio $\frac{\triangle \mathbf{B}}{\triangle x}$ is also present due to the essentially nonlinear dependence of the field magnitude on distance ($|\mathbf{B}| \sim 1/|\mathbf{r}|^3$) inevitably increases as approach the field source. In the same context, consideration of the possibility of using scalar sensors to construct a vector gradiometer deserves special attention. High-precision and highly sensitive scalar quantum magnetometers with optical pumping could be used with a small distance between them in the structure of the installation, but their design is such that bringing the sensors closer to each other than 1.5 m radically distorts the readings. No less important factors that can destroy the harmonious scheme of basic algorithms are other imperfections of various magnetically sensitive sensors and the measurement system as a whole: orientation errors, various types of nonlinearities, temperature drift of zeros and scale factors, etc.

6. EXPERIMENTS TO EVALUATE THE ACCURACY OF DETERMINING THE RELATIVE POSITION

The above features of the technical implementation make the possibility of putting the basic algorithms into action not entirely obvious and explain the desire to conduct experiments on the actually achievable capabilities of the system in terms of: the required characteristics of the sensors and the measurement system as a whole, the available range of distances between the source and the field reciever, the degree of influence of various types of interference, the potentially achievable accuracy of determining geometric parameters, and speed of operation. To assess the technical feasibility and confirm the effectiveness of the considered algorithms, a series of experiments was carried out, the task of which was to assess the accuracy of determining distances and directions in real conditions, taking into account natural magnetic interference and the limited accuracy of magnetically sensitive sensors, as well as the limited accuracy of monitoring the dipole magnetic moment of the emitter.

A loop transmitter was used as a field source—a flat inductor with a diameter (500 mm, 100 turns), fed by a meander-shaped current with a frequency of 4 Hz. The amplitude of the

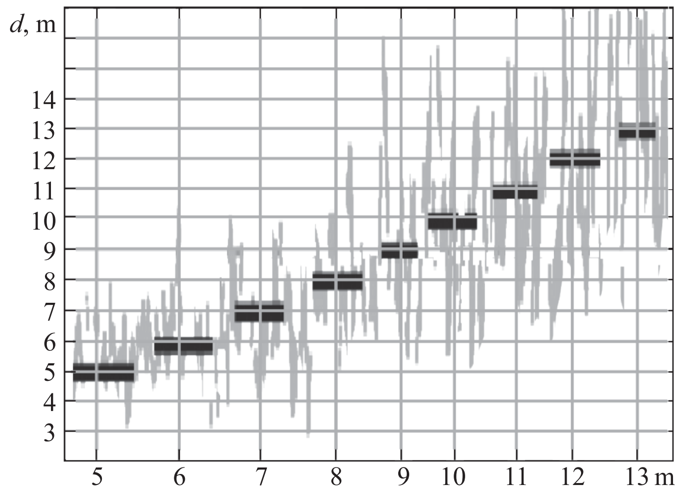

Fig. 7. Calculating the distance to the field source.

dipole magnetic moment was about 35 Am^2; to simplify control, the direction of the vector was set horizontal. The tensor-type magnetic gradiometry reciever was composed of three vector fluxgate magnetometers HB0302 [7], having a sensitivity of 1.0–5.0 nT. The sensors were installed on a rotating platform in a horizontal plane along the vertices of an equilateral triangle with an edge length of 1.0 m. The experiments were preceded by a series of calibration procedures, the coverage of the theoretical foundations and technology of which is beyond the scope of this article. The sequence of measurement procedures in the final experimental design was presented in the following series.

At a known distance from the center of the triangle of the magnetic gradiometry system to the field source (this distance ranged from 5 m), a series of measurements were performed in which the magnetic gradiometry measurement installation remained stationary, and the loop transemitter, maintaining its location in space, sequentially changed the direction of the dipole moment in azimuth. Then the measurement installation, remaining in place, changed its position in azimuth. This series made it possible to evaluate the accuracy of determining the direction to the source and the direction of the vector of its dipole moment. Measurements in this sequence were performed twice. The first part were as the basis for calibration procedures, and according to the data of the other, accuracy control was carried out.

The second series of experiments consisted of monitoring the accuracy of determining the distance to the field source for different directions of the dipole magnetic moment vector. The magnetic measurement installation remained stationary, and the loop transmitter with a step of 2.0 m moved away from the reciever at a distance of 5 to 13 m. At each position, four measurements were performed at different directions of the dipole magnetic moment vector. In this series, the accuracy of determining the distance to the source was assessed at various distances and for various directions of the dipole moment vector.

During the experiments, the following results were obtained.

Figure 7 shows the results of an experiment to determine the distance to the source from gradient measurements. The specified values of the distance between the dipole-transmitter and the measurement installation are plotted horizontally, and calculated values are displayed vertically. The curve shows the calculated value, the horizontal segments—averaged for each of the time intervals corresponding to the distance of the dipole from point to point with a step of 1.0 m.

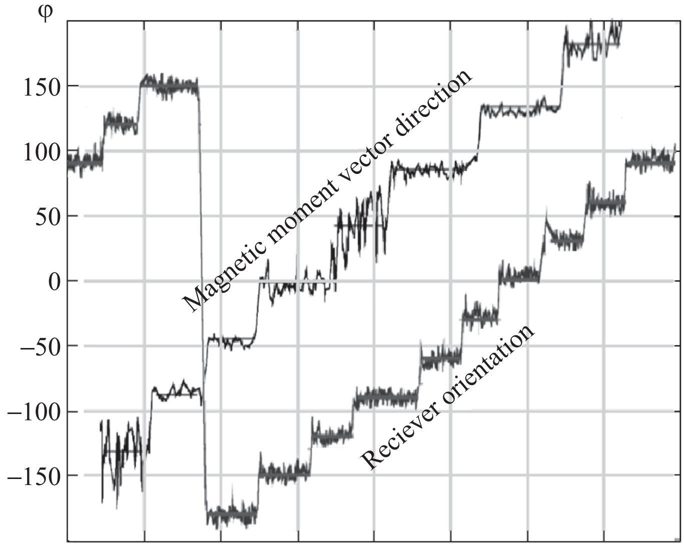

Fig. 8. Calculation of angular orientation parameters.

It can be seen from the figure that the distance to the source in the presented experimental design is generally calculated reliably. The small discrepancy is explained by the imperfection of the experimental conditions: the significant influence of magnetic interference in the measurement area, as well as the error in the placement of the transmitter dipole relative to the measurement system. The resulting accuracy in this experiment was 4–9% depending on the value of the determined distance.

Figure 8 shows two graphs showing the possibility of determining the parameters of the mutual angular orientation of the receiving system and transmitter from magnetic gradient measurements. The graphs show the results of changes over time in determining the values of the angles of the azimuthal orientation of the dipole magnetic moment vector (direction of the moment vector) and the selected axis of the measurement installation (orientation of the meter). The calculated values are plotted along the vertical axis. Line segments in the graphs show preset values. It is clearly seen from the figure that in this experiment the direction to the dipole-transmitter was determined based on the results of magnetic gradiometry measurements in general more accurately than the direction of the dipole moment vector, however, taking into account the simplicity of the measurement scheme, in general, sufficient reliability of the operation of the algorithms for determining both directions is shown.

The resulting accuracy in determining the orientation of the reciever was 3–10° depending on the distance. The resulting accuracy in determining the direction of the dipole moment vector depends not only on the distance, but also on the orientation of the reciever. It was 10–30° depending on the distance.

7. CONCLUSION

The research presented in the paper made it possible to formulate the basic principles of a promising method of relative angular and spatial positioning of objects. The above calculations show the fundamental possibility of constructing structurally and functionally simple high-precision systems useful for solving problems of controlling the movement of objects during their interaction: mooring, docking, refueling in the air, monitoring the position of the ship relative to the anchor, etc. The experiments presented in this work confirmed the perfomance capabilities of constructing systems operating on the principles of the algorithms discussed in the work.

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